Condition for the eqn

$$ax^2 + 2hxy + by^2 + 2fx + 2gy + c = 0$$

to represent two lines

$$by^2 + 2(hx+g)y + (ax^2 + 2fx + c) = 0$$

$$y = \frac{-2(hx+g) \pm \sqrt{4(hx+g)^2 - 4b(ax^2+2fx+c)}}{2b}$$

$$\text{Discriminate} = h^2x^2 + 2ghx + g^2 - abx^2 - 2fbx - bc = c.s$$

$$= (h^2 - ab)x^2 + 2(hg - bf)x + (g^2 - bc) = c.s$$

$$4(hg-bf)^2 - 4(h^2-ab)(g^2-bc) = 0$$

OR $$\Delta = \begin{vmatrix} a & h & f \\ h & b & g \\ f & g & c \end{vmatrix} = 0$$

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EX : Find the two lines, θ, and the eqns of the bisectores for the eqn $2y^2 - 7xy + 10x^2 + 7x + y - 12 = 0$

$(y - 2x + \alpha)(y - 5x + \beta) = 0$

Comparing the coefficients we got :

C.O.X : $-5\alpha - 2\beta = 7 \longrightarrow 1)$

C.O.y : $\alpha + \beta = 1 \longrightarrow 2)$

$2\alpha + 2\beta = 2$

$-3\alpha = 9$

$\alpha = -3$

$\beta = 4$

∴ المعطيين هما :-

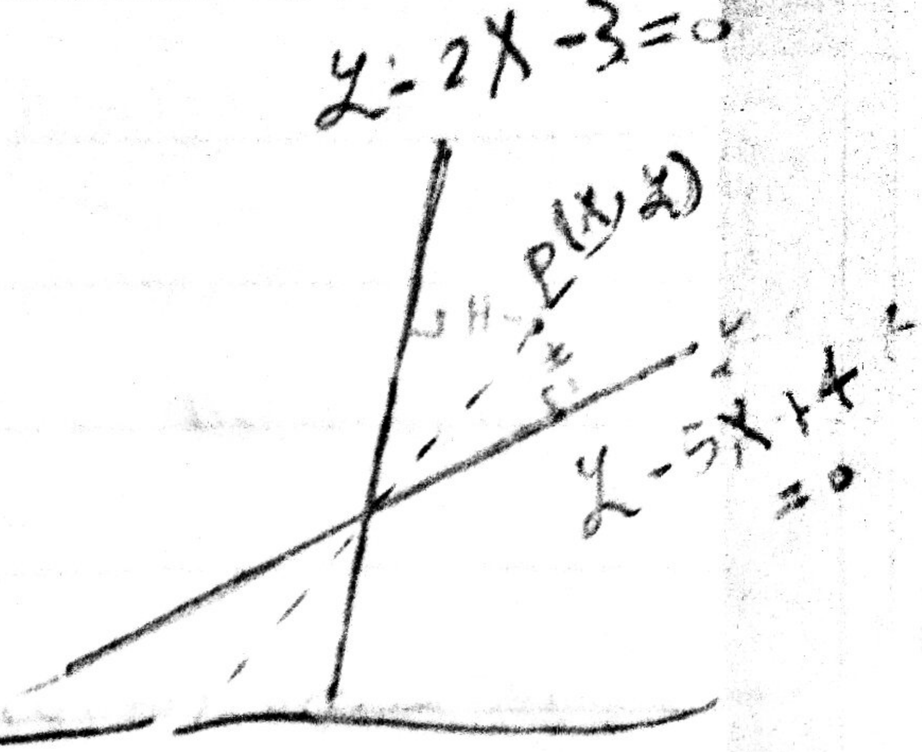
$y - 2x - 3 = 0$ & $y - 5x + 4 = 0$ #

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b} = \frac{3}{11}$$

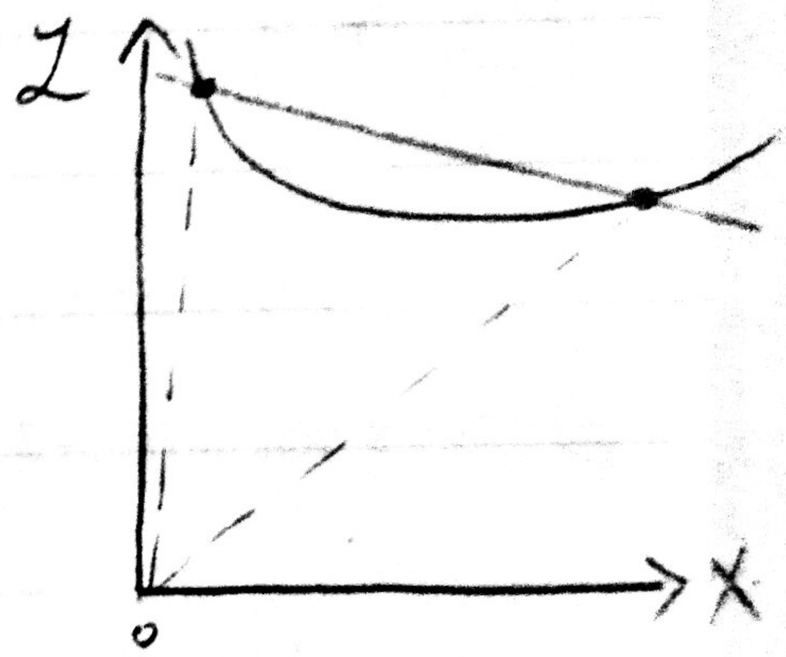
$\theta = 15.255°$

Date

$$\frac{2-2X-3}{\sqrt{1+4}} = \pm \frac{2-5X+4}{\sqrt{1+25}}$$



Ex: Find the eqn of two line thro and the pts of intersection of $aX^2 + 2hXY + bY^2 + 2fX + 2gY + C = 0$ and the line $a_1X + b_1Y = C_1$



Sol

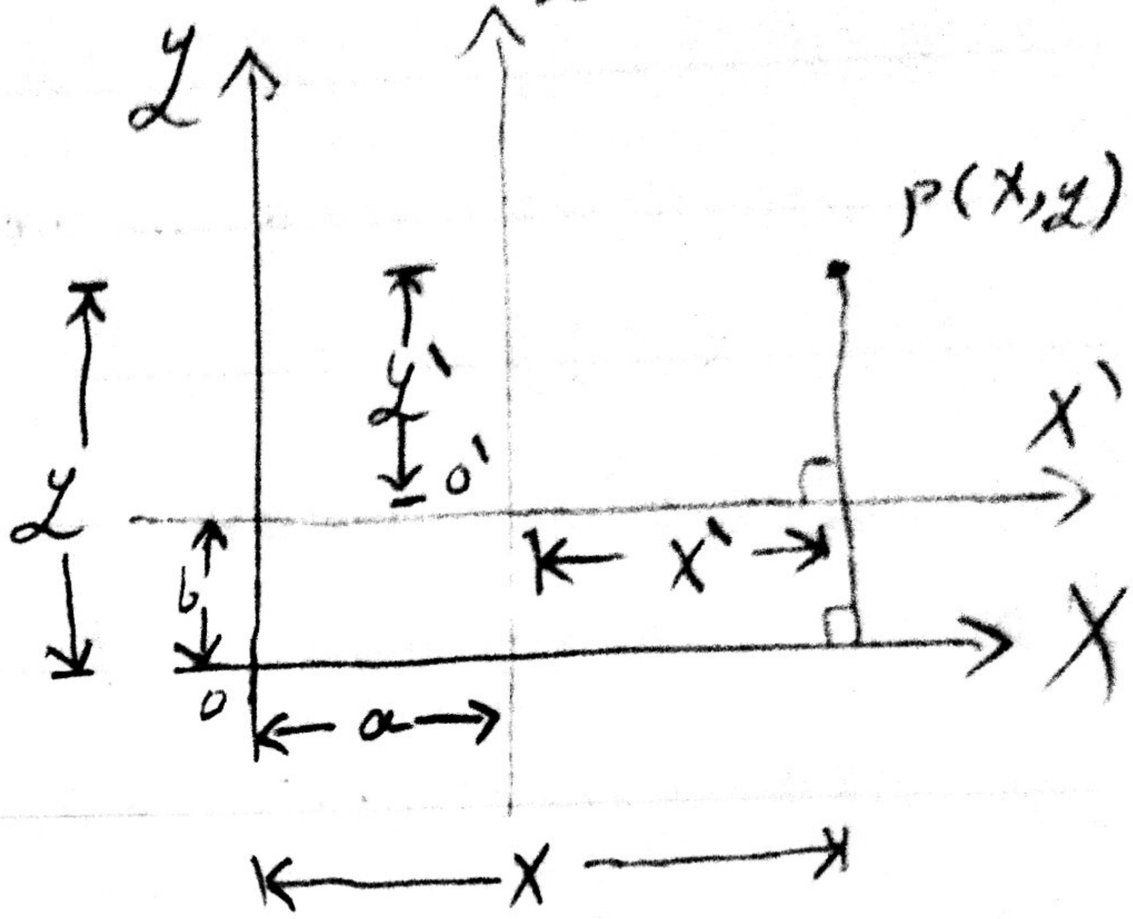
$$\frac{a_1X + b_1Y}{C_1} = 1$$

$$\equiv$$

$$aX^2 + 2hXY + bY^2 + 2(fX+gY)\left(\frac{a_1X+b_1Y}{C_1}\right)^{\times(1)} + C\left(\frac{a_1X+b_1Y}{C_1}\right)^{2\times(1)} = 0 \quad \#\#$$

Translation & Rotation of axes :-

(1) Translation

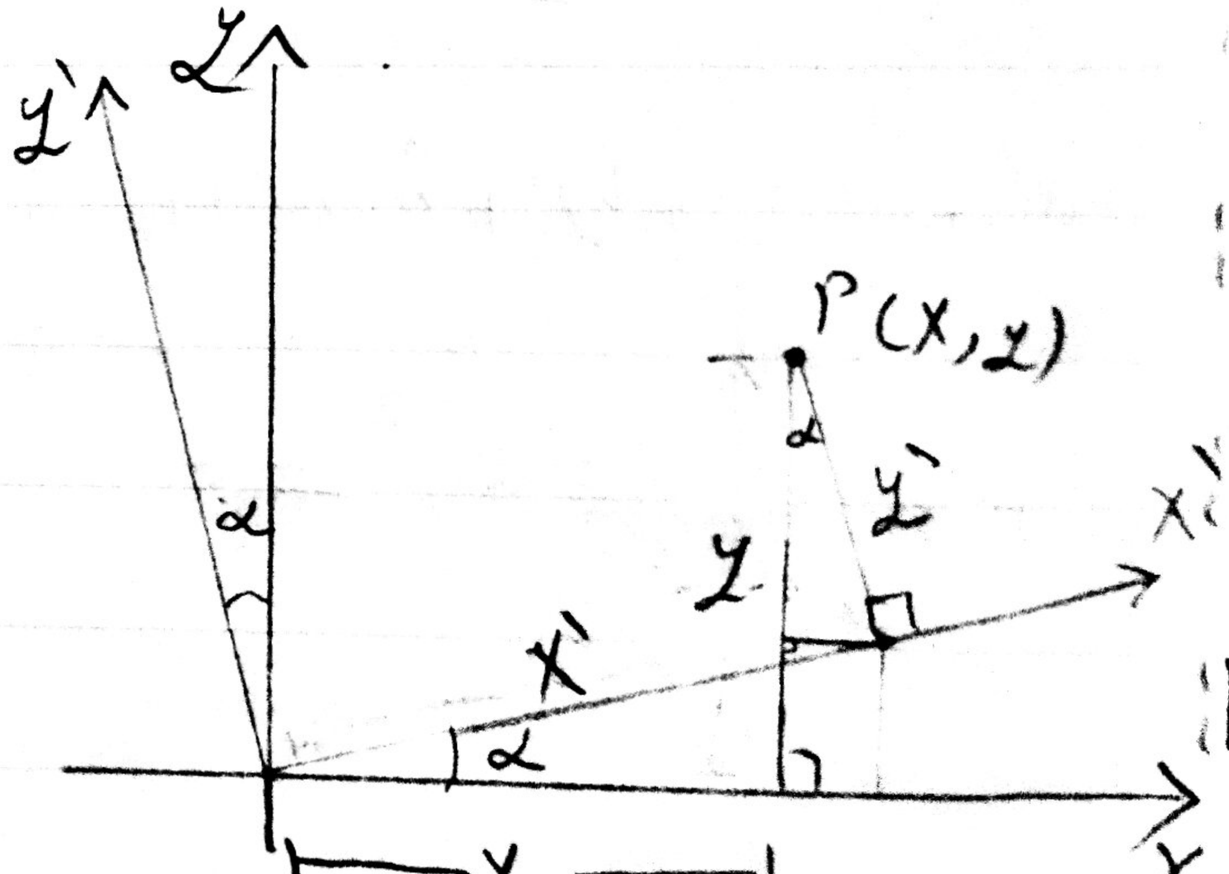


$o(0,0) \longrightarrow o'(a,b)$

$X = X' + a$

$Y = Y' + b$

(2) Rotation of axes



$X = X'\cos\alpha - Y'\sin\alpha$

$Y = X'\sin\alpha + Y'\cos\alpha$

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} X' \\ Y' \end{pmatrix}$$

$$\begin{pmatrix} X' \\ Y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

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EX: Eliminate X, Y from the eqn $X^2+5XY+2Y^2-13X-22Y+26=0$

transsrate to the point (a, b)

Let $X = X' + a$

$Y = Y' + b$

$(X'+a)^2 + 5(X'+a)(Y'+b) + 2(Y'+b)^2 - 13(X'+a) - 22(Y'+b) + 26 = 0$

C.o X': $2a + 5b - 13 = 0 \quad \rightarrow 1)$

C.o.Y': $5a + 4b - 22 = 0 \quad \rightarrow 2)$

$a = 2 \quad , b = 3$

NOTE: